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SIMPLIFIED MODELING OF LIQUID-STRUCTURE INTERACTION IN THE SEISMIC ANALYSIS OF CYLINDRICAL LIQUID STORAGE TANKS

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SUMMARY

The present study presents simplified closed form solution to be used to model the effect of liquid hydrodynamic pressure in the linear and non-linear analysis of cylindrical-vertical liquid storage tanks. The added mass approach was used to derive the added mass index equation. The equation may then be used to compute the added mass matrix prior to the analysis in lieu of modeling the liquid domain inside the tank. Cases were presented where the results obtained from the analysis of the fully coupled liquid-structure models were compared to the results obtained from the analysis of simplified models using the developed added mass equation. It was concluded that the developed equation provides accurate enough results for the purpose of the structural design of such tanks, their foundations and connections.

INTRODUCTION

It is common in the earthquake engineering practice to encounter the problem where it is required to perform seismic analysis of a vertical-cylindrical liquid storage tank. The current design standards and specifications do not provide accurate estimation of the seismic demand and capacity on liquid storage tanks, specially the unanchored ones, Haroun [2] and [3]. The finite element method is a good alternative method to provide accurate analysis of such tanks because of the tremendous advancements of the computer technology coupled with the advancements of easy-to-use commercial structural-engineering-oriented finite element packages. All these easy-to-use finite element packages do not provide the user of the ability to model the fully coupled liquid-structure interaction problem. Only few advanced, expensive and hard to use commercial and research-oriented packages provide this feature. The present study provides the alternative, which enables the user of the easy-to-use finite element packages to analyze these tanks taking into consideration the effect of liquid-structure interaction. The simplified added mass index equation presented in this study may be used to calculate the added mass at every node on the wet surface of cylindrical-vertical tanks. The user may then apply these added masses to the tank to simulate the hydrodynamic effect of the contained liquid.

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THEORETICAL BACKGROUND OF THE ADDED MASS MATRIX

The hydrodynamic pressure of the liquid domain in a tank is governed by the following pressure wave equation written for incompressible fluids

$$\nabla^2 p(x,y,z,t) = 0 \tag{1}$$

Where *p*(*x*,*y*,*z*,*t*) is the pressure distribution in the liquid domain. Following the approach used in Kuo [6], this strong form leads to the following weak form

$$\iiint_V \nabla \mathbf{N}^T \cdot \nabla \mathbf{p} \, dV = r \iint_S \mathbf{N}_S^T \cdot \mathbf{n} \cdot \ddot{\mathbf{u}} \, dA \tag{2}$$

Where **N** is a vector of shape functions, $\mathbf{N}_s$ is the vector of shape functions at the liquid-structure wet surface, *r* is the mass density of the liquid, *V* is the volume of the liquid domain, *A* is the liquid-structure wet surface area, **n** is the normal vector to the wet surface at a generic point and $\ddot{\mathbf{u}}$ is the acceleration vector at a generic point on the wet surface. Proceeding with the finite element formulation leads us to the following expression for the added mass matrix as shown in Kuo [6]

$$\mathbf{M} = r \; \mathbf{h}_\mathbf{S}^\mathbf{T} \mathbf{g}_\mathbf{S}^{-1} \mathbf{h}_\mathbf{S} \tag{3}$$

Where,

$$\mathbf{h}_\mathbf{S}^\mathbf{T} = \bigcup_{\text{All Elements}} \iint_S \mathbf{N}_S^T \cdot \mathbf{n} \cdot \mathbf{N}_S \; dA \tag{4}$$

$$\mathbf{g} = \bigcup_{\text{All Elements}} \iiint_V \left[\frac{\partial \mathbf{N}^T}{\partial x}\frac{\partial \mathbf{N}}{\partial x} + \frac{\partial \mathbf{N}^T}{\partial y}\frac{\partial \mathbf{N}}{\partial y} + \frac{\partial \mathbf{N}^T}{\partial z}\frac{\partial \mathbf{N}}{\partial z} \right] dV \tag{5}$$

The symbol U denotes assembly over all elements. The matrix $\mathbf{g}_\mathbf{s}$ is given by the expression

$$\mathbf{g}_\mathbf{s} = \mathbf{g}_\mathbf{ss} - \mathbf{g}_\mathbf{sr}^{-1} \, \mathbf{g}_\mathbf{rr} \; \mathbf{g}_\mathbf{rs} \tag{6}$$

Where $\mathbf{g}_\mathbf{ss}$, $\mathbf{g}_\mathbf{sr}$, $\mathbf{g}_\mathbf{rr}$ and $\mathbf{g}_\mathbf{rs}$ are the sub-matrices of the matrix **g** partitioned into two groups of degrees of freedom, the first group denoted by r includes the degrees of freedom that are neither on the liquid-structure surface nor on the free surface and the second group denoted by s includes all degrees of freedom that are on the liquid-structure wet surface but not on the free surface.

LUMPING TECHNIQUE OF THE ADDED MASS MATRIX

The added mass matrix **M** is a full matrix. For many reasons, this represents an obstacle before including the numerical modeling of the liquid-structure interaction in easy-to-use structural engineering oriented finite element software. Among these reasons are the following:

(i) The fact that full matrices do not allow the efficient banded solvers and banded storage to be implemented

(ii) The size of the full added mass matrix is too large to be manually entered

(iii) Lack of accommodation of such models in the easy-to-use finite element programs

Thus, the only solution to this problem is to lump the added mass matrix at every node on the wet surface of the tank. The full added mass matrix was first calculated and then transformed to the cylindrical coordinates as follows

$$M_{ij} = \left\langle n_i^T \right\rangle \cdot \left[M_{ij} \right] \cdot \left\{ n_j \right\} \tag{7}$$

where $\mathbf{n}_i$ and $\mathbf{n}_j$ are the normals to the liquid boundary at nodes i and j, respectively, M_{ij} is the added mass at node i in $\mathbf{n}_i$ direction due to the acceleration of node j in $\mathbf{n}_j$ direction, and $[M_{ij}]$ is the mass submatrix of nodes i and j in the Cartesian coordinates.

The lumped mass element at each node is then calculated as follows

$$M_i^* = \sum_{j=1}^{no} M_{ij} \ \vec{n}_i \cdot \vec{n}_j \tag{8}$$

Where no is the total number of nodes on the wet surface of the structure. The added mass index (I) is then calculated by dividing the lumped added mass element at each node by the tributary area of that node. In other words, given the mass index at node i (I_i), the lumped added mass submatrix at this node in the x, y and z directions is given as follows

$$\begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix}_{at\ node\ \ i} = I_i \ A_i \begin{bmatrix} n_x n_x & n_x n_y & n_x n_z \\ n_y n_x & n_y n_y & n_y n_z \\ n_z n_x & n_z n_y & n_z n_z \end{bmatrix}_{at\ node\ \ i} \tag{9}$$

Where A_i is the tributary area of node i. This procedure was implemented for many tanks of different heights, diameters and aspect ratios. The least square approximation method was then used to perform curve fitting of the results. The following closed form expression for the mass index on the walls of cylindrical-vertical liquid storage tanks was established

$$I(y,H,R) = r\ H\left[0.8 + 0.26\frac{y}{H} - 0.96(\frac{y}{H})^2\right] \ \tanh\left(1.4\frac{R}{H}\right) \tag{10}$$

Where y is the height of the node measured from the base, H is the height of the liquid in the tank and R is the radius of the tank. The expression of the mass index for the base plate is found to be equal to the liquid density times the liquid height, which is the total mass of the column of liquid of unit base area and height H.

COMPARISON WITH FULLY COUPLED LIQUID-TANK MODEL

In order to assess the accuracy of the developed closed form expression for the mass index, the period of the first Ritz vector for anchored liquid storage tanks of different heights and aspect ratios was calculated and compared to the period of the first Ritz vector of the corresponding fully coupled three dimensional liquid-structure model, Haroun [2], as well as the period of the first mode of the impulsive mode of the mechanical analog of anchored tanks, Haroun [4]. The first Ritz vector was calculated due to a horizontal acceleration field applied on the tank in the direction of the x-axis. The modeled tanks are assumed to be full of liquid. A linear constraint is imposed on the nodes located on the circle at the top of the tank wall, at the intersection of the wall with the roof, to simulate the effect of the roof. This constraint will force all nodes located on that circle to move together as a rigid body, i.e. the circle representing the intersection between the tank roof and the tank wall remains a circle at all times and does not deform under any loading condition.

The sample study case presented here is an anchored tank of 60 feet height, 30 feet radius and has a wall thickness of 1 inch. Figure (1) shows the deformed shape of the first Ritz vector of the tank using the full-added mass matrix. This Ritz vector is the vector that will govern the response of such tanks during seismic events. The same tank was then analyzed using the lumped added mass index approach, calculated using Equations (9) and (10), and similar shape of the Ritz vector was calculated and the corresponding frequency was found to be 6.44 Hz as compared to 6.42 Hz for the case of full-added mass matrix. To verify these values, the frequency of the first impulsive mode of the mechanical analog developed by Haroun was calculated to be 6.40 Hz. Table (1) documents the comparison of several other case studies.

The present study shows that the first Ritz vector dominating the seismic response of liquid storage tanks is not sensitive to the off-diagonal distribution of the mass matrix. The reason behind that could be clearly understood from Figure (2), which shows the distribution of the mass index of the sample study case. This distribution represents the change in hydrodynamic pressure at tank nodes due to the acceleration of the node indicated in the figure, which is one column in the full matrix of the added mass index. For the purpose of obtaining clear plots of contour lines, the walls of the tank were rotated around the base to fall in the same plane as the tank base plate. It is clear from these contour lines that most of the added mass is concentrated around the accelerated node itself with little effect over other nodes.

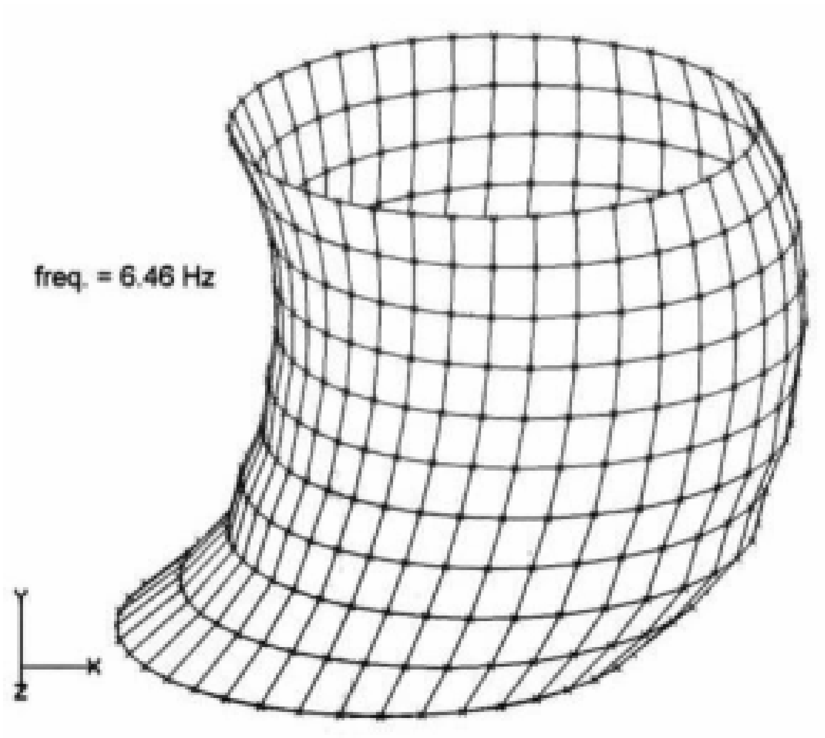

Figure (1): Deformed Shape of First Ritz Vector of Tank Using Full Added Mass Matrix

CONCLUSION

A simplified closed form solution of the added mass matrix to the walls and base plate of cylindrical-vertical liquid storage tanks was found. Comparisons with the finite element analysis of fully coupled liquid-structure finite element models show that the derived expression is accurate enough for the purpose of designing the tank, and its foundation and connections. This expression can be easily incorporated into commercial easy-to-use finite element packages to model the hydrodynamic pressure on cylindrical liquid storage tanks.

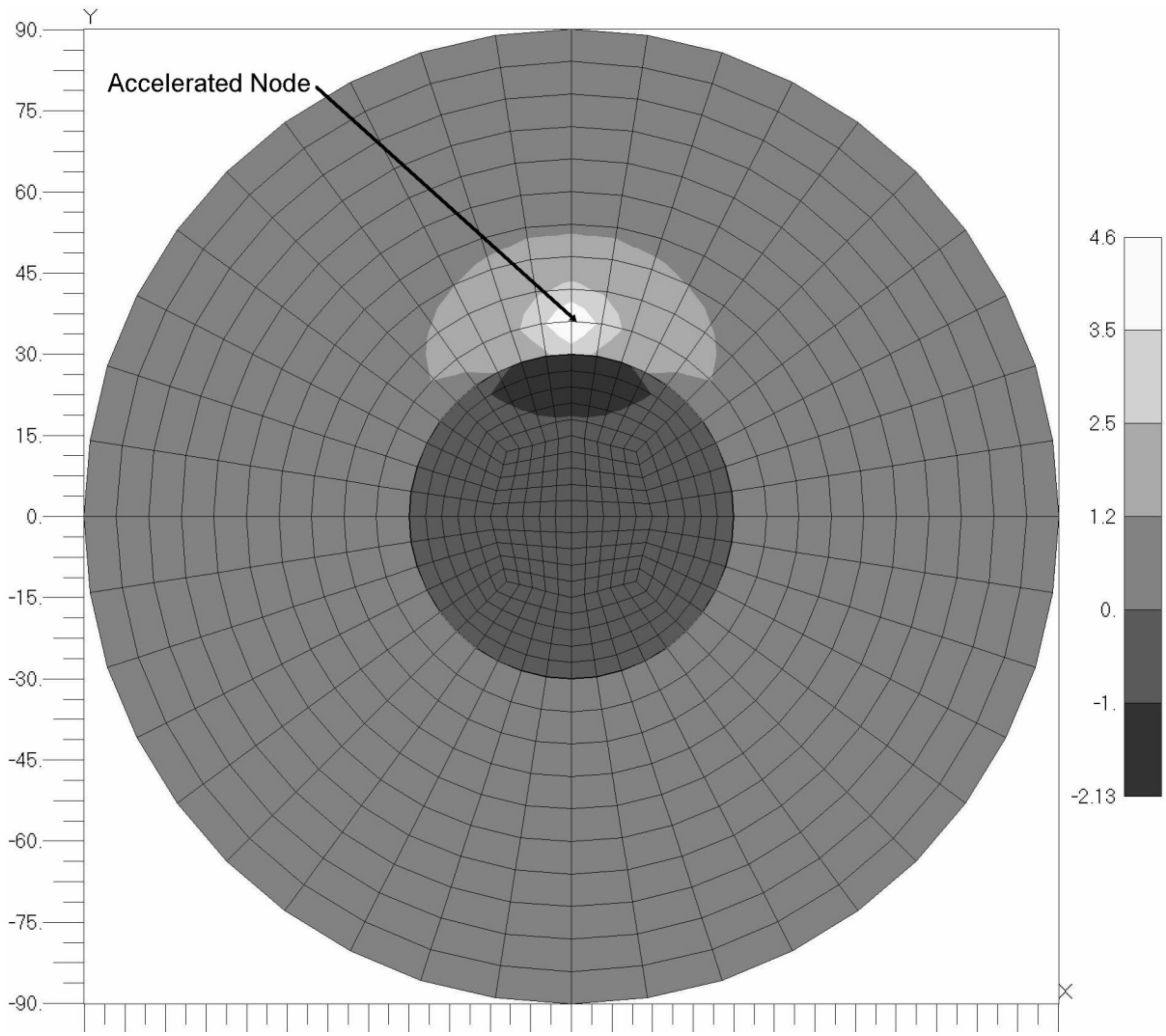

Figure (2): Distribution of the Mass Index of the Sample Study Case

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Table (1): Comparison of the Frequency (Hz) of the First Ritz Vector Calculated using Different Techniques. All Cases Studied have 1 inch Base Plate and Shell Thickness

Tank Height (feet)	Tank Radius (feet)	Full-Added Mass (Hz)	Lumped Added Mass (Hz)	Haroun's Mechanical Analog (Hz)
40.0	60.0	6.61	6.28	6.10
40.0	50.0	7.05	6.75	7.01
40.0	40.0	7.82	7.51	8.23
40.0	30.0	9.19	8.92	9.81
40.0	20.0	11.75	11.71	11.56
50.0	75.0	4.74	4.50	4.36
50.0	62.5	5.06	4.83	5.03
50.0	50.0	5.61	5.39	5.90
50.0	37.5	6.60	6.41	7.05
50.0	25.0	8.46	8.43	8.36
60.0	90.0	3.61	3.43	3.32
60.0	75.0	3.85	3.68	3.83
60.0	60.0	4.27	4.11	4.50
60.0	45.0	5.04	4.89	5.38
60.0	30.0	6.46	6.44	6.39