NONLINEAR TRANSIENT RESPONSE OF UNANCHORED LIQUID STORAGE TANKS

Medhat A. Haroun, Professor and Chair
and
Al Zeiny, Graduate Research Assistant
Department of Civil and Environmental Engineering
University of California, Irvine, U.S.A.

ABSTRACT

Commonly used seismic analysis methods for unanchored liquid storage tanks have repeatedly demonstrated inconsistencies in predicting the actual behavior of these structures. Most methods incorporate a pseudo-dynamic analysis in lieu of the full time-dependent seismic analysis. Further, the estimation of liquid-exerted overturning moments lacks explicit consideration of the support effects. These overturning moments tend to lift the shell off its foundation which may cause buckling of the shell on one side and a separation of the base plate from the shell on the other. A finite element program capable of handling the complexities associated with the nonlinear transient response of unanchored tanks was developed. It was observed that the overturning moment exerted on an unanchored tank may be smaller than that exerted on a similar anchored tank, yet the stresses at the shell bottom of an unanchored tank shell are generally much larger than those of an anchored tank subjected to same ground motion.

INTRODUCTION

The present study was carried out to investigate the effects of transient liquid-exerted hydrodynamic pressures on thin-walled unanchored liquid storage tanks during earthquake motions. Complexities arise due to the successive contact and separation between the base plate and its foundation, nonlinear fluid-structure interaction, large-amplitude deformations of the base plate, pre- and post-buckling behavior of the shell, material yielding, soil-tank interaction, and large-amplitude free-surface sloshing.

A number of studies was reported in the literature on investigations of the seismic behavior of unanchored tanks. Due to the complexity of the problem, most of the original studies were experimental in nature. Several simplified theoretical investigations were also conducted and a few of these studies have been used as a basis for current design standards. Yet, the large-scale damage to unanchored tanks in recent earthquakes highlighted the need for a careful analysis of such tanks.

Numerical discretization approaches using the finite element method or the finite difference method have been employed recently to analyze unanchored tanks. However, assumptions of varied degrees of approximations were made to simplify the analysis, such as the substitution of the base plate by "equivalent" springs, the performing of a pseudo-dynamic analysis in lieu of the full dynamic analysis, the linearization of a portion of the problem such as considering the tank wall to be rigid or ignoring liquid sloshing, and the use of approximate analytical expressions for the hydrodynamic pressures to eliminate the liquid degrees of freedom.

The present study employs the finite element technique to analyze unanchored tanks taking into consideration base plate contact with a flexible foundation and its large-amplitude deformations, buckling behavior of the shell, material yielding and large-amplitude free-surface sloshing. A three-dimensional fully coupled liquid-structure model was subjected to a seismic ground motion and the time history response of various design parameters was obtained. In addition, a comparison between the earthquake response of anchored and unanchored tanks is presented.

FUNDAMENTAL APPROACH

A computer model capable of simulating the complex transient behavior of unanchored liquid storage tanks, when subjected to strong seismic base excitations, was developed. The model takes into consideration large amplitude liquid sloshing and the geometric, material and contact nonlinearities of the tank shell and base plate. The computer simulation included the following features:

- A variational principle that forms the basis for the numerical discretization of fully-coupled nonlinear liquid-structure interaction problems with free surface sloshing. The program uses an updated Eulerian-Lagrangian description of the liquid-structure interface in order to enforce compatibility between structure and liquid elements.

- An up-to-date finite element technology in the analysis of structures and curved shells using the degeneration concept, and considering both material plasticity and geometric nonlinearity.

- Free surface sloshing modeling that utilizes the nonlinear wave theory formulation. The updated Lagrangian description of the liquid domain boundaries is utilized to keep track of the free surface position at any time.

- The foundation is modeled using tensionless springs. This approach was found to be efficient in representing the nonlinear uplift problem.

- An efficient handling of the contact/uplift analysis of unanchored tanks. A Lagrange multiplier technique was employed to enforce both displacement compatibility and force transmissibility constraints along the unknown contact surface.

- The nonlinear governing equations are solved using an efficient time integration technique that has been developed specifically to solve liquid-structure interaction problems.

VARIATIONAL PRINCIPLES

The dynamic response of liquids has significant influence on the response of their containers. Inappropriate approximation of the liquid motion may lead to major errors in estimating the seismic response of the container. The liquid pressures and the impact forces form the measurable level of the energy transferred to the tank shell. In addition, the motion of the tank wall is a primary source for the liquid energy. Since this energy transfer occurs simultaneously throughout the liquid boundary, it is essential in the finite element analysis of such problems to use a model that effectively deals with the coupling between the liquid and the tank wall.

The equations of motion of a liquid may be formulated by two different approaches. The Eulerian formulation is obtained by tracking the velocity, pressure and density for all points of the space occupied by the liquid at all instances. The Lagrangian formulation is obtained by considering the history of each particle. In the current investigation, a Lagrangian description of the structure's motion is utilized, which makes it necessary to use a Lagrangian description of the liquid-structure interface in order to enforce compatibility between the structure and the liquid elements. The continuity equation in the Eulerian form is utilized inside the liquid domain to mathematically describe the liquid motion inside the tank.

The liquid in this analysis is considered to be inviscid, irrotational and incompressible. Such simplifying assumptions allow displacements, pressures or velocity potentials to be the variables in the liquid domain. The displacement-based liquid elements may be easy to incorporate in many finite element programs for structural analysis and it may simplify the enforcement of the liquid-structure interface constraints. However, such elements require two or three degrees of freedom per node. In addition, this approach is not well suited for problems with large liquid displacements and requires special care to prevent zero-energy rotational modes. Alternatively, using pressures or velocity potentials as the unknown degrees of freedom requires only one degree of freedom per node inside the liquid domain, which significantly reduces the computational cost of the analysis, yet adequately represents the physical behavior of the liquid. The latter approach is used in this investigation.

Structure Domain

The virtual work statement of the structural domain may be written as

$$\begin{aligned}\delta\Pi_s &= \int_{\Omega_s} \delta\epsilon^T \mathbf{E}\, \epsilon \, d\Omega_s + \int_{\Omega_s} \rho_s \delta \mathbf{u}^T \ddot{\mathbf{u}} \, d\Omega_s \\ &- \int_{\Gamma_w} \delta\mathbf{u}^T \mathbf{f}^I \, d\Gamma_w - \int_{\Omega_s} \delta\mathbf{u}^T \mathbf{f}^E \, d\Omega_s = 0 \quad (1)\end{aligned}$$

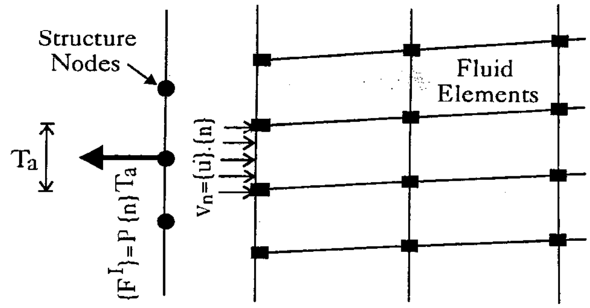

Figure 1: Boundary conditions at a structure node in contact with liquid element

where $\mathbf{E}$ is the stress-strain matrix, ϵ is the strain vector, $\mathbf{u}$ is the displacement vector, Ω_s is the structural domain, Γ_w is the wet surface of the structure, $\mathbf{f}^I$ is the liquid pressure vector, $\mathbf{f}^E$ is the body force vector and ρ_s is the mass density of the structure.

Liquid Domain

Following the work done by Kock and Olson (1991), the variational indicator of an incompressible liquid flowing under gravity field is obtained as

$$\Pi_l = \int_{t_1}^{t_2} \int_{\Omega_l} \left\{ \rho_l g y + \frac{1}{2}\rho_l \nabla^2 \phi + \rho_l \dot{\phi} \right\} d\Omega_l dt \qquad (2)$$

or concisely,

$$\Pi_l = \int_{t_1}^{t_2} \int_{\Omega_l} P d\Omega_l dt \qquad (3)$$

where P is the total pressure which may be also written as

$$P = P_o - \gamma_l \left[\frac{1}{g}\frac{\partial \phi}{\partial t} + \frac{\nabla\phi . \nabla\phi}{2g} + y \right] \qquad (4)$$

where P_o is the hydrostatic pressure at the point, ρ_l is the mass density of the liquid, y is the Cartesian coordinate measured in a direction opposite to that of the gravitational acceleration g, and ϕ is the velocity potential.

Coupled Liquid-Structure System

In order to apply the variational principle to the liquid-structure interaction problem, the liquid and the structure functionals, given by Eq. (1) and (3), are added. The two statements are coupled at the liquid-structure interfaces by

$$\dot{u}_n = \frac{\partial \phi}{\partial \mathbf{n}} \qquad (5)$$

$$\mathbf{f}^I = P\,\mathbf{n} \qquad (6)$$

where $\mathbf{n} = (n_x, n_y, n_z)$ is the outward normal unit vector from the liquid towards the structure. Figure (1) illustrates the interaction between a structure node with a liquid element, where T_a is the tributary area of the structure node.

NUMERICAL IMPLEMENTATION

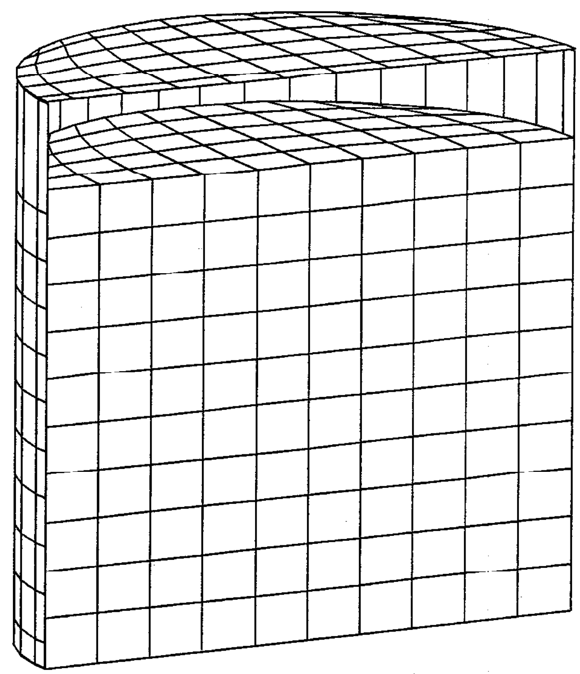

Figure 2: Finite element mesh of the coupled liquid-tank system

The preceding basic approach was incorporated in a nonlinear transient finite element program DYNAZ (El-Zeiny, 1995). The program was used to analyze an unanchored tank located at the American National Can site (ANC) which, during the 1994 Northridge earthquake, experienced severe ground motion (Haroun, 1995). The tank suffered elephant foot buckling at the bottom of the shell. It apparently uplifted during the earthquake severing the adjacent piping and releasing its contents. The tank is 37 ft in diameter, 32 ft in height and has a shell of 3/8 in thickness. It is supported on a concrete pad and was filled with water to a depth of 28.3 ft at the time of the earthquake. The tank was subjected to the record measured at the Arleta site which has a peak ground acceleration of $0.344g$.

As shown in Fig. (2), the liquid inside the tank was discretized to 500 fluid elements with a total of 726 nodes. The tank was discretized to 359 isoparametric

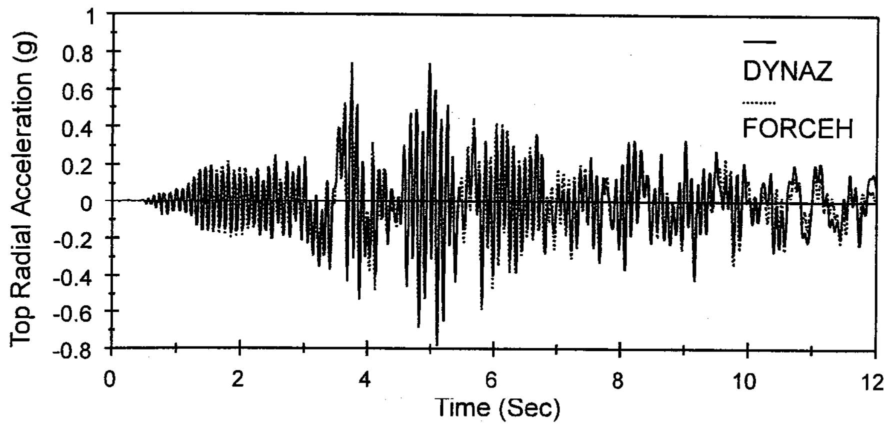

Figure 3: Horizontal acceleration at shell top

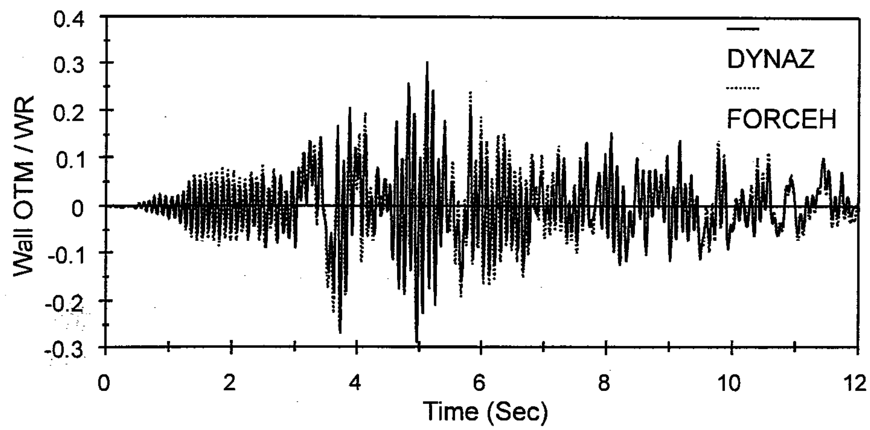

Figure 4: Base overturning moment

Parameter	Anchored	Unanchored
Total Overturning Moment	0.407 WR	0.320 WR
Liquid Overturning Moment	0.383 WR	0.245 WR
Overturning Moment on Wall	0.303 WR	0.170 WR
Total Base Shear	0.473 W	0.276 W
Base Shear due to Liquid	0.454 W	0.257 W
Maximum Shell Acceleration	0.88 g	1.96 g
Maximum Uplift Displacement	–	1.22 in (3.10 cm)
Minimum Contact Area	–	73.2%
Axial Stress at Shell Bottom	-2.19 ksi (15.1 MPa)	-5.27 ksi (36.3 MPa)
Hoop Stress at Shell Bottom	1.45 ksi (10.0 MPa)	11.61 ksi (80.1 MPa)
Axial Stress at Shell Mid-height	-0.54 ksi (3.7 MPa)	-1.00 ksi (6.9 MPa)
Hoop Stress at Shell Mid-height	5.83 ksi (40.2 MPa)	5.42 ksi (37.4 MPa)

Table 1: Comparison Between Anchored and Unanchored Tank Responses

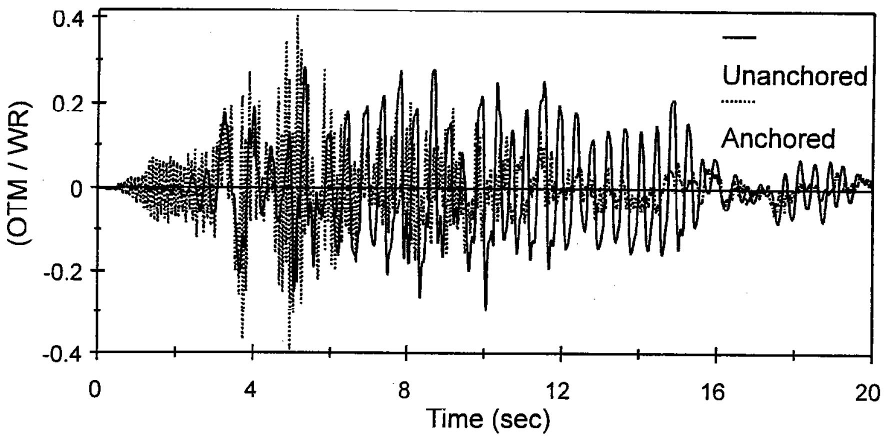

Figure 5: Overturning moment about the center of base of the ANC tank

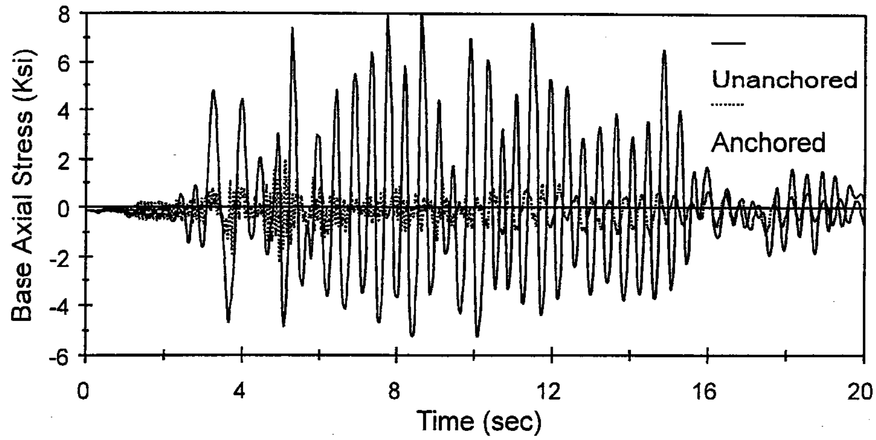

Figure 6: Base axial stress in the ANC tank shell

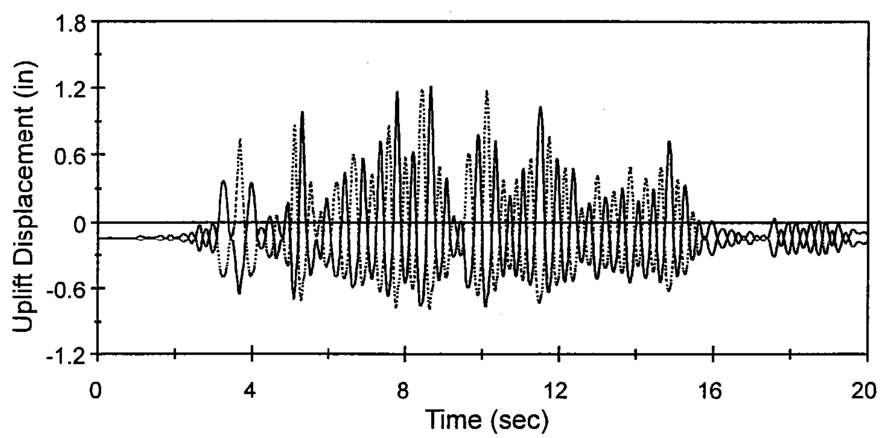

Figure 7: Uplift displacement of the two corner points on the principal diameter

shell elements with a total of 428 nodes. The finite element formulation simulated both the liquid-structure interaction and the free surface sloshing, and large-deflection theory was used to account for buckling. Two different boundary conditions for the tank were considered: anchored tank condition in which the tank base plate is considered fully fixed to a rigid foundation, and the unanchored tank condition in which the tank base plate is considered supported on a tensionless elastic foundation of a uniform stiffness of 100 $lb/in/in^2$ in compression.

In a previous work (Haroun, 1981), the anchored tank response was estimated by the computer code FORCEH which uses the combined finite element and boundary solution techniques. The elastic shell was modeled by finite elements and the fluid region was treated analytically as a continuum. This combined approach is advantageous in that the number of unknowns is substantially less than the model which discretizes both the tank shell and the liquid as three-dimensional objects. However, the former solution is restricted to a linear analysis and cannot be used to analyze unanchored tanks. To assess the accuracy of the present program, a comparison of the anchored tank responses using the two approaches is presented. Figure (3) shows the time history response of the horizontal acceleration of the top node of the tank shell. Figure (4) illustrates the time history of the base overturning moment due to the pressure on the tank wall. Clearly, close correlation was found between the two approaches.

The ANC unanchored tank was analyzed by the program DYNAZ. Table (1) shows a comparison between the tank responses under the anchored and unanchored conditions. Note that W denotes the total weight of the contained liquid, R is the tank radius and g is the acceleration of gravity. The response of the unanchored tank was governed primarily by a rocking motion. This mode was found to have a dominant period of 0.4 sec. and, based on this period, the foundation rocking damping is estimated to be 3%. On the other hand, the response of the anchored tank was governed primarily by the flexible-impulsive pressure component which has a fundamental period of 0.09 sec. A Raleigh damping coefficient which provides 3% damping to the first mode and increasing values for the higher modes was chosen for the flexible-impulsive component. Since the rocking period is relatively large as compared to the flexible-impulsive period, the overturning moment exerted on the anchored tank was found to be larger than that exerted on the unanchored tank. However, due to the nature of the boundary conditions associated with the base of the unanchored tank, the axial and hoop stresses at the bottom of the unanchored tank shell were much larger than those of the anchored tank. Figures (5) and (6) show a time history comparison between the anchored and the unanchored tank for the overturning moment measured at the center of the base plate and for the base axial stresses, respectively.

The contact characteristics of the unanchored tank with its foundation are important factors in evaluating the response of such a tank. Figure (7) shows the uplift displacement of the two extreme opposite points on the principal diameter which parallels the earthquake excitation. It shows that the uplift displacement on the tension side is much higher than the penetration displacement on the compression side. Such a behavior is expected due to the tensionless nature of the foundation. In addition, Fig. (8) shows the time history of the change in the area of contact of the base plate with the foundation as compared to the total area. It should be noted that the results presented here include the effects of contact and geometric nonlinearities. The effect of material nonlinearities, considered by the formation of plastic hinges, increases the uplift displacement (El-Zeiny, 1995). Similar observations were noted for other unanchored tanks under different earthquake excitations.

CONCLUSION

A finite element program capable of analyzing the complexities associated with the nonlinear dynamic response of unanchored liquid storage tanks was developed. It was observed that the overturning moment exerted on an unanchored tank may be smaller than that exerted on a similar anchored tank due to the longer-period nature of the rocking-motion which dominates the behavior of unanchored tanks. However, due to the nature of the boundaries associated with unanchored tanks at their base, the axial and hoop stresses at the bottom of an unanchored tank shell may be much larger than those of an anchored tank subjected to the same ground motion.

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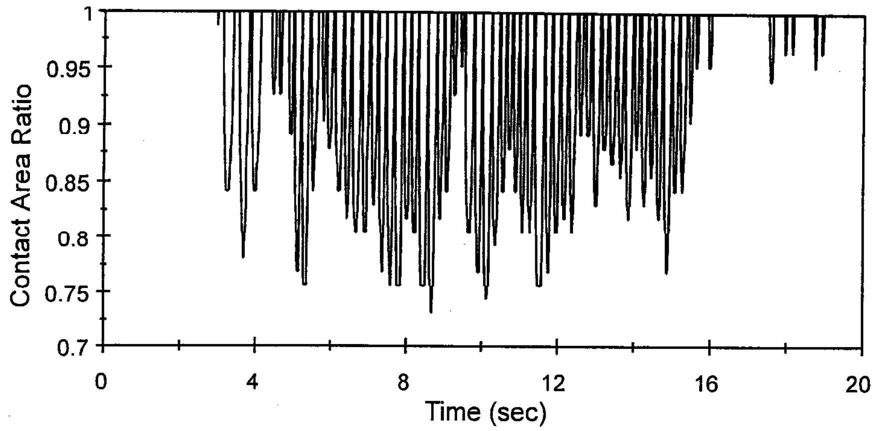

Figure 8: Percentage of bottom plate area in contact with the foundation

REFERENCES

Barton, D., and Parker, J., *Finite Element Analysis of the Seismic Response of Anchored and Unanchored Liquid Storage Tanks*, Earthquake Engineering and Structural Dynamics, Vol. 15, 1987, pp. 299-322.

Zeiny, A.A., *Nonlinear Dynamic Analysis of Liquid Storage Tanks*, Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of California, Irvine, 1995.

Haroun, M.A., *Earthquake Response of Deformable Liquid Storage Tanks*, Journal of Applied Mechanics, ASME, Vol. 48, No. 2, June 1981, pp. 411-418.

Haroun, M.A., and Zeiny, A.A., *Seismic Design Considerations for Uplifted Tanks,* Structural Engineering in Natural Hazards Mitigation, ASCE, Irvine, Vol. 1, April 1993, pp. 139-144.

Haroun, M.A., Abou-Izzeddine, W., and Modé, N., *Comparative Evaluation of Seismic Design Standards of Oil Storage Tanks,* Proceedings of the Pressure Vessels and Piping Conference, Vol. 272, ASME, Minneapolis, Minnesota, June 1994, pp. 51-58.

Haroun, M.A., and Bhatia, H., *Analysis of Tank Damage During the 1994 Northridge Earthquake,* Proceedings of the Fourth US Conference on Lifeline Earthquake Engineering, San Francisco, August 1995.

Ishida, K., Kobayashi, N., and Harima, I., *An Effective Method of Analyzing Rocking Motion for Unanchored Cylindrical Tanks Including Uplift*, Pressure Vessels and Piping Conference, ASME, Vol. 98-7, New Orleans, June 1985, pp. 87-96.

Kock, E., and Olson, L., *Fluid Structure Interaction Analysis by the Finite Element Method - a Variational Approach*, International Journal for Numerical Methods in Engineering, March 1991, Vol. 31, pp. 463-491.

Malhotra, P.K., and Veletsos, A.S., *Uplifting Response of Unanchored Liquid-Storage Tanks*, Proceedings of the Journal of Structural Engineering, Vol. 120, December 1994, pp. 3525-3547.

Natsiavas, S., *Simplified Models for the Dynamic Response of Tall Unanchored Liquid Containers*, Pressure Vessels and Piping Conference, ASME, Vol. 157, 1989, pp. 15-21.

Sakai, F., Isoe, A., Hirakawa, H., and Montani, Y., *Experimental Study on Uplift Behavior of Large-sized Cylindrical Liquid Storage Tanks*, Pressure Vessels and Piping Conference, ASME, Vol. 127, 1989, pp. 349-355.

Wozniak, R.S., and Mitchell, W.W., *Basis of Seismic Design Provisions for Welded Steel Oil Storage Tanks*, Session on Advances in Storage Tank Design, API, Toronto, Canada, May 1978.